



**Swallowdale
Primary School**

Calculation Policy

2023/2024

This calculation policy has been reviewed and developed from the Bradgate Education Partnership and from White Rose calculation policies as a tool to support the teaching and learning of calculation in KS1 and KS2 at Swallowdale Primary School.

Introduction

This Calculation Policy has been written with the aims of the National Curriculum at the heart of it.

The National Curriculum Aims

The national curriculum for mathematics aims to ensure that all pupils:

- ♣ become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately
- ♣ **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- ♣ can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions

Each of the four operations build on a solid understanding of place value, the connections between the four number operations and number sense, such as: whether they are odd or even, whether they are close to multiples of ten or if they are close together.

- Children need to use correct mathematical terminology in context and be able to verbalise their calculation strategies.
- Children need to make considered decisions as to the most appropriate methods to make mathematics more functional. They need to choose the most appropriate, fluent, efficient and accurate method to do a particular calculation.
- Children need to use concrete resources before they progress to pictorial and abstract representations. This CPA (concrete, pictorial and abstract) approach needs to be available to children throughout school, as and when necessary. Use of manipulatives (numicon, dienes, HTO counters etc.) helps reinforce understanding and provides support when calculating mentally, mentally with jottings, using expanded methods and formal written methods. Use of the bar model, number lines and part- part whole diagrams are recommended.

- Children should progress between the stages working towards formal written methods (where appropriate), once they have mastered each stage. However, they should not be hurried and, after the method has been taught, children should still be able to make their preferred choice of the most appropriate, efficient and accurate method for them. Previous stages may need to be revisited to consolidate understanding when introducing a new strategy.
- As new methods of calculations are introduced, children should have the opportunity to examine them, alongside the method they have consolidated, to make connections between the methods and establish the similarities and differences between them.

This policy includes sections on: Addition, Subtraction, Multiplication and Division. It outlines progression in teaching, from mental through to formal written methods.

Addition

Models and Images

number tracks
bead strings
number lines [marked and unmarked]
Base 10
place value counters
place value (arrow) cards
ten frames
Numicon
counting sticks
hundred squares
bar model
IT resources
commutative law

Key Vocabulary

add	sum
addition	altogether
plus	increase
and	more than
count on	count all
total	

Fluency

retrieval

number fact retrieval

make connections with real life such as
adding number of hot dinners and
sandwiches together

addition games and quizzes

songs and rhymes

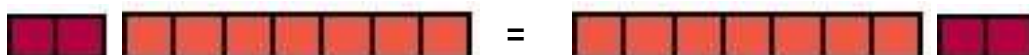
questions for continuous provision

Year 1 Objectives

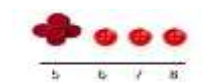
- Given a number, identify one more.
- Read, write and interpret mathematical statements involving addition (+) and the equals (=) sign.
- Add one-digit and two-digit numbers within 20, including zero.
- Solve missing number problems (e.g. $6 + \underline{\quad} = 10$).

Mental Strategies

- Show how the commutative law can be used to reorder numbers when adding e.g. put the larger number first: **$2 + 7$ becomes $7 + 2$**



- Count on in ones or twos: **$5 + 3$ becomes $5 + 1 + 1 + 1$**



Children will practise counting on from any number: 'Put five in your head and count on three.' Initially, use a number track to count on for addition, counting from the largest number, progress to using number lines.



- Partition small numbers: **$6 + 5$ becomes $6 + 4 + 1$**



- Partition and combine tens and ones: **$12 + 7$ becomes $10 + 2 + 7$**

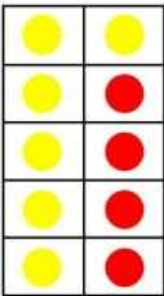
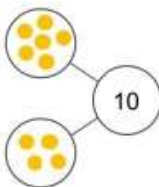
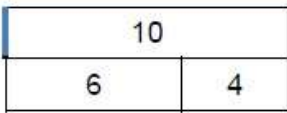


- Use number facts knowledge: **$7 = 7 + 0$ or $6 + 1$ or $5 + 2$, etc.**

- Number bonds to 10 and 20: **$5 + 5$; $7 + 3$; $15 + 5$; $16 + 4$, etc.**



- Use concrete and pictorial representations to solve missing number problems:

 $10 = 4 + 6$ $10 = 6 + 4$ $6 = 10 - 4$ $4 = 10 - 6$ Tens Frame	 $6 + 4 = 10$ $4 + 6 = 10$ $10 - 4 = 6$ $10 - 6 = 4$ Part Whole Model	 $6 + 4 = 10$ $4 + 6 = 10$ $10 - 4 = 6$ $10 - 6 = 4$ Bar Model
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Year 2 Objectives- continue to build on from Year 1

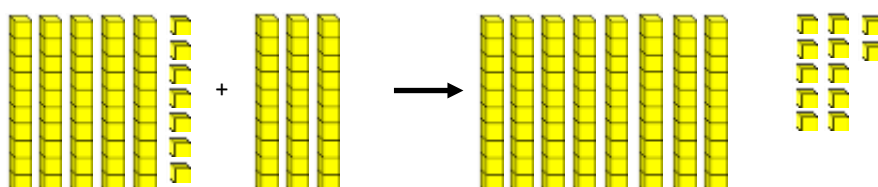
- a two digit number and ones;
- a two digit number and tens;
- two two-digit numbers;
- three one-digit numbers.

Mental Strategies

- Use place value and say 1 more and 10 more than any two digit number

	1	2	3	4	5	6	7	8	9
11	12	2	3	4	5	6	7	8	9
21	22	32	33	34	35	36	37	38	39
47	48	49	40	41	42	43	44	45	46
67	62	63	64	65	66	67	68	69	60
54	55	56	57	58	59	50	51	52	53
61	62	63	64	65	66	67	68	69	60
19	27	28	29	30	31	32	33	34	35
72	73	74	75	76	77	78	79	80	81
82	83	84	85	86	87	88	89	90	91
92	93	94	95	96	97	98	99	00	01

- Use place value to partition: $57 + 35$ as $50 + 30 = 80$ then $7 + 5 = 12$ then $80 + 12 = 92$



- Use place value to partition then adjust: **$45 + 9$ becomes $45 + 10 = 55$ then $55 - 1 = 54$**
 $45 + 21$ becomes $45 + 20 = 65$ then $65 + 1 = 66$
- Count on in multiples of 10: **$76 + 20$ as $76 + 10 + 10$ saying 76, 86, 96**
- Use patterns of known facts: **$6 + 3 = 9$, so $36 + 3 = 39$ and $76 + 3 = 79$**
- Bridge through 10: **$57 + 5$ as $57 + 3 + 2 = 62$**
- Use number bonds to 10 knowledge when adding 3 or more single digit numbers:
 $4 + 7 + 6$ as $10 + 7$

concrete



pictorial



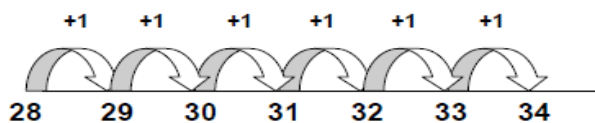
abstract

$$\begin{aligned} (4 + 7) + 6 &= 10 + 7 \\ &= 17 \end{aligned}$$

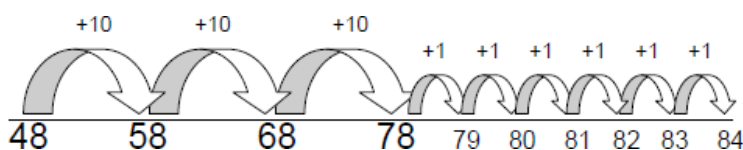
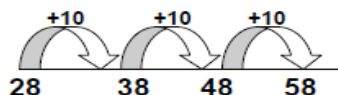
Written Strategies

Count on in ones and tens, using an empty number line, within 100:

$$28 + 6 = 34$$



$$28 + 30 = 58$$

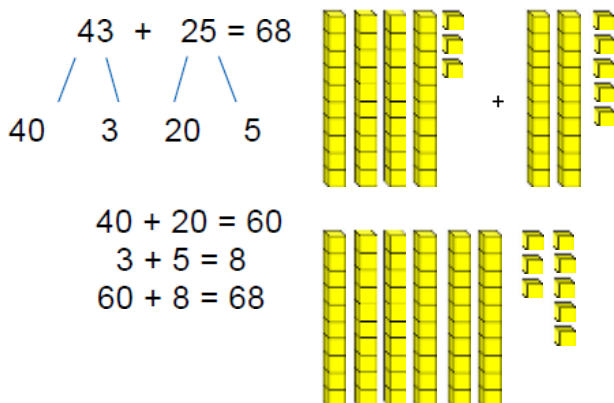


$$48 + 36 = 84$$

Partition into tens and ones when adding.

Also, use the partitioning method to add two two-digit numbers.

- Partition the numbers into tens and ones.
- Add the ones together and then add the tens together.
- Recombine to give the answer.
- Then move on to calculations that bridge the tens:



$$48 + 36 = 40 + 8 + 30 + 6$$

$$40 + 30 = 70$$

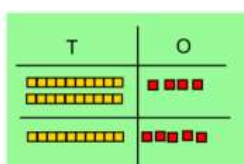
$$8 + 6 = 14$$

$$70 + 14 = 84$$

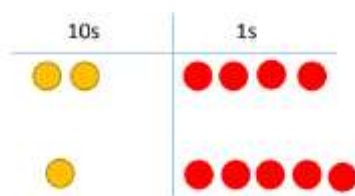
$$48 + 36 = 84$$

Use column method for adding without regrouping

concrete



pictorial



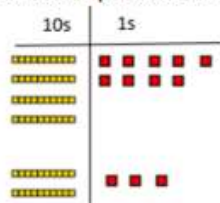
abstract

$$\begin{array}{r} 24 \\ + 15 \\ \hline 39 \end{array}$$

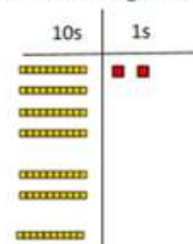
Column method with regrouping

concrete

Make both numbers on a place value grid.

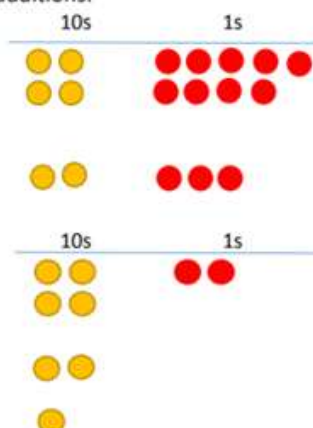


Add up the units and exchange 10 ones for 1 ten.



pictorial

Using place value counters, children can draw the counters to help them to solve additions.



abstract

$$\begin{array}{r} 40 + 9 \\ 20 + 3 \\ \hline 60 + 12 = 72 \end{array}$$

Year 3 Objectives- continue to build on from Year 2

Add and subtract numbers mentally, including:

- a three-digit number and ones;
- a three-digit number and tens;
- a three-digit number and hundreds.

Add numbers with up to three digits, using formal written method of columnar addition, where appropriate.

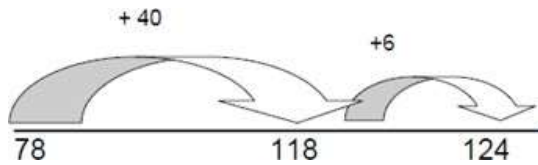
Mental Strategies

- Using place value to partition and count on / add ones, tens and hundreds.
- Use number lines to support visualisations.

Written Strategies

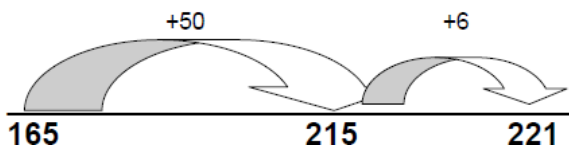
Use Year 2 progression smaller steps to start then further develop the use of the empty number line with calculations in more efficient steps that bridge 100:

$$78 + 46 = 124$$



... and with addition of a three-digit and a two-digit number:

$$165 + 56 = 221$$



Further develop the partitioning method with calculations that bridge 100:

$$85 + 37 = 80 + 5 + 30 + 7$$

$$5 + 7 = 12$$

$$80 + 30 = 110$$

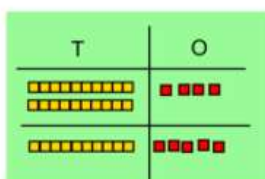
$$110 + 12 = 122$$

$$85 + 37 = 122$$

The partitioning method can also be used with three-digit numbers

Column method without regrouping further developed from Year 2

concrete



pictorial



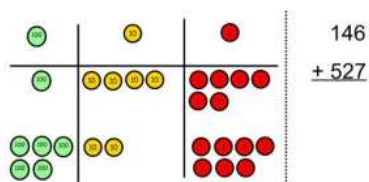
abstract

$$\begin{array}{r} 24 \\ + 15 \\ \hline 39 \end{array}$$

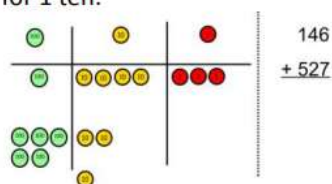
column method with regrouping

concrete

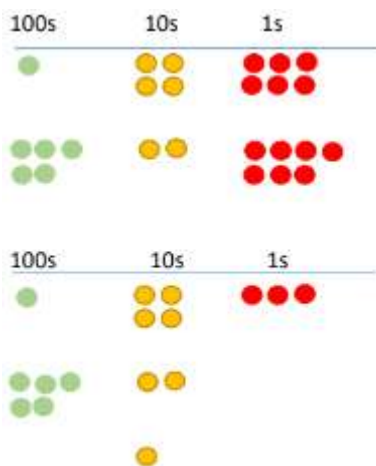
Make both numbers on a place value grid.



Add up the units and exchange 10 ones for 1 ten.



pictorial



abstract

$$\begin{array}{r} 100 + 40 + 6 \\ 500 + 20 + 7 \\ 600 + 70 + 3 = 673 \end{array}$$

As the children progress, they will move from the expanded to the compacted method.

$$\begin{array}{r} 1 \\ 146 \\ + 527 \\ \hline 673 \end{array}$$

Year 4 Objectives – continue to build on from Year 3

Add and subtract numbers mentally, including:

- a four-digit number and ones;
- a four-digit number and tens;
- a four-digit number and hundreds;
- a four-digit number and thousands.

Add and subtract numbers with up to 4 digits, using formal written method of columnar addition, where appropriate.

Written Strategies

Use empty number lines with three and four digit numbers, as appropriate.

Further develop the formal written method of addition with three-digit numbers, revisiting the expanded method first, if necessary.

$$\begin{array}{r} 176 + 147 = 323 \\ 1 \quad 1 \\ 1 \quad 4 \quad 7 \\ + 1 \quad 7 \quad 6 \\ \hline 3 \quad 2 \quad 3 \end{array}$$

Year 5 Objectives – continue to build on from Year 4

Add and subtract numbers mentally, with increasingly large numbers.

Add whole numbers with more than 4 digits, including using formal written method (column addition).

Mental strategies

- Using place value to partition and count on, including decimals.
- Using estimating and rounding strategies.

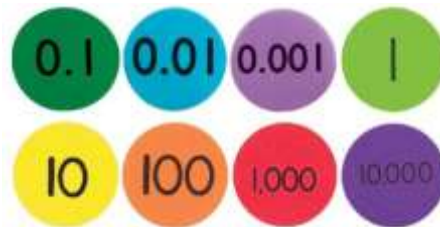
Written Strategies

Use the formal written method for the addition of decimal numbers:

$$154.75 + 233.82 = 388.57$$

$$\begin{array}{r} 1 \\ 233.82 \\ +154.75 \\ \hline 388.57 \end{array}$$

Place value counters should be used to support these calculations:



Ensure that the decimal points line up. Continue to use the language of place value to ensure understanding.

Year 6 Objectives- build on learning from previous year groups

Perform mental calculations, including with mixed operations and large numbers.

. Children should continue to practise and use formal written method for larger numbers and decimals.

Our aim is that, by the end of Y6, children use mental methods (with jottings) when appropriate, but for other calculations, they use an efficient, formal written method accurately and with confidence.

Subtraction

Models and Images

number tracks
bead strings
number lines [marked and unmarked]
Base 10
place value counters
place value (arrow) cards
ten frames
hundred squares
bar model
IT resources

Key Vocabulary

subtract	less than
take away	fewer than
take from	decrease by
distance between	deduct
difference	reduce
count back/on	minus
inverse	rename

Children need to understand that subtraction is not commutative or associative.

Fluency

daily games
retrieval questions
true or false questions
problem of the day
songs and rhymes

27 children here today I have 18 reading diaries how many am I missing?

Year 1 Objectives

- Given a number, identify one less.
- Read, write and interpret mathematical statements involving subtraction (-) and the equals (=) sign.
- Subtract one-digit and two-digit numbers within 20, including zero.
- Solve missing number problems e.g. $7 = \square - 9$

Mental Strategies

- Children will practise counting back from any number: 'Put seven in your head and count back two.'



- Use a number track or bead strings to count back for subtraction, counting back from the largest number, progressing to using number lines.

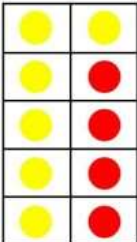
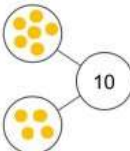
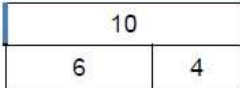


- Use pictures to take away ones $4 - 2 = 2$
- Use number bond facts to 10 and 20: $10 - 5 = 5$ $7 = 10 - 3$ $15 = 20 - 5$
- Use concrete and pictorial representations to solve missing number problems:

$$17 - \square = 4$$

		$17 = 13 + 4$	$4 + 13 = 17$
		$17 - 4 = 13$	$17 - 13 = 4$

- Use the inverse of addition to help with subtraction.

 $6 + 4 = 10$ $4 + 6 = 10$ $10 - 4 = 6$ $10 - 6 = 4$	 $6 + 4 = 10$ $4 + 6 = 10$ $10 - 4 = 6$ $10 - 6 = 4$	 $6 + 4 = 10$ $4 + 6 = 10$ $10 - 4 = 6$ $10 - 6 = 4$
Tens Frame	Part Whole Model	Bar Model

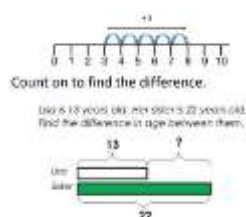
- Find the difference

concrete

Compare amounts and objects to find the difference.



pictorial



abstract

Hannah has 8 goldfish.
Helen has 3 goldfish.
Find the difference between the number of goldfish the girls have.

Year 2 Objectives- continue to build on from Year 1

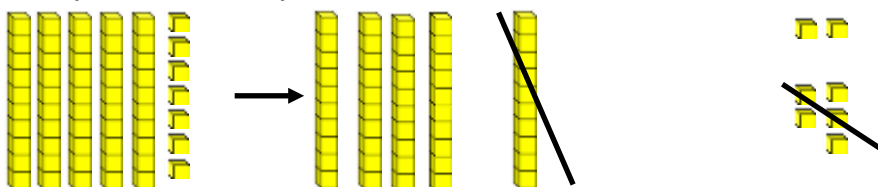
- a two digit number subtract ones;
- a two digit number subtract tens;
- two-digit numbers subtract two-digit.

Mental Strategies

- Use place value and say 1 less and 10 less than any two digit number

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- Use place value to partition:: **57-15 is 7-5 = 2 then 50-10 = 40 then 40 + 2 = 42**



- Use place value to partition then adjust:

45 - 9 becomes 45 - 10 = 35 then 35 + 1 = 36

45 - 21 becomes 45 - 20 = 25 then 25 - 1 = 24

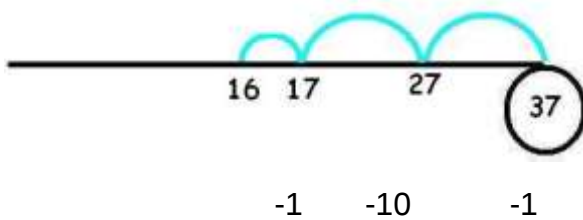
- Count back in multiples of 10: **76 - 20 is 76 - 10 - 10 saying 76, 66, 56**
e.g. using hundred squares / base ten / number lines
- Use patterns of known facts: **6 - 3 = 3, so 36 - 3 = 33 and 76 - 3 = 73**
- Use known facts to bridge through 10: **57 - 9 as 57 - 7 - 2 = 48**
e.g. use tens frames / base ten / number line to represent.

●	●	●	●	●
●	●	●	●	

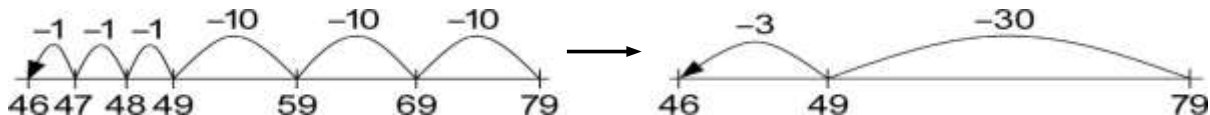
Written Strategies

- Count back in ones and tens within 100 (e.g. using an empty number line)

$$37 - 21 = 16$$



- Begin to use more efficient jumps: $79 - 33 = 46$



- Partition the numbers into tens and ones.
- Subtract the ones first and then subtract the tens.
- Recombine to give the answer.

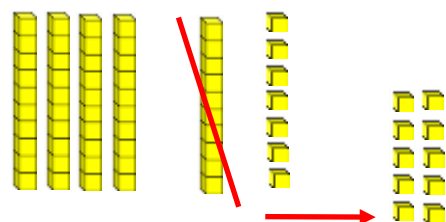
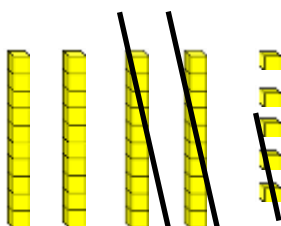
- Then move on to calculations that bridge the tens – Exchange:

e.g. $57 - 19 = 38$

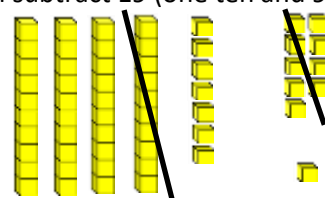
$$\begin{array}{r} 45 \\ - 23 \\ \hline 22 \end{array}$$

40 5 20 3

$$\begin{aligned} 40 - 20 &= 20 \\ 5 - 3 &= 2 \\ 20 + 2 &= 22 \end{aligned}$$



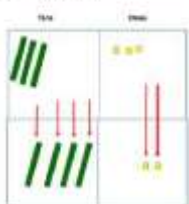
- Exchange** a ten stick for 10 ones
- (This example shows 57 becoming 40 + 17)
- Then subtract 19 (one ten and 9 ones)



Column method without regrouping

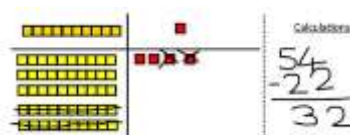
concrete

$$75 - 42 = 33$$



Use Base 10 to make the bigger number then take the smaller number away.

pictorial



abstract

$$\begin{array}{r} 54 \\ - 22 \\ \hline 32 \end{array}$$

Year 3 Objectives- continue to build on from Year 2

Subtract numbers mentally, including:

- a three-digit number subtract ones;
- a three-digit number subtract tens;
- a three-digit number subtract hundreds.

Subtract numbers with up to three digits, using formal written method of columnar subtraction, where appropriate.

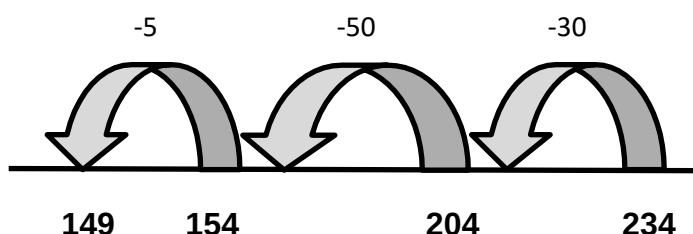
Mental Strategies

- Using place value to partition and subtract ones, tens and hundreds.
- Use number lines to support visualisations.

Written Strategies

- Further develop the use of the empty number line with calculations that bridge 100:

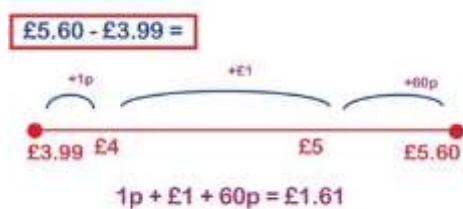
$$234 - 85 = \boxed{}$$



NB: You may wish to support the subtraction of the multiple of ten by showing this in jumps of ten to begin with.

- Develop subtraction in a different context: $£5.60 - £1.61 = \boxed{}$

- Find the difference by counting on: $£5.60 - \boxed{} = £3.99$



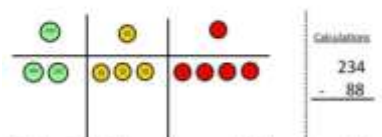
Column method without regrouping- Build from Year 2

$$\begin{array}{r} 63 \\ - 32 \\ \hline 31 \end{array}$$

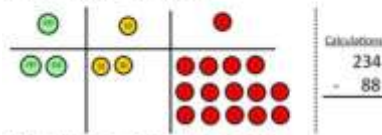
Column method with regrouping

concrete

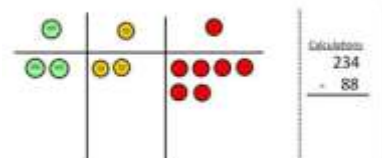
Make the larger number with the place value counters



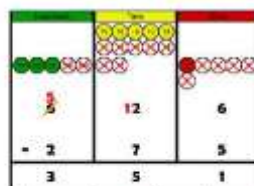
Start with the ones, can I take away 8 from 4 easily? I need to exchange 1 of my tens for 10 ones.



Now I can subtract my ones.

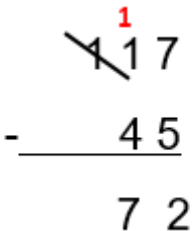


pictorial



Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.

abstract



Year 4 Objectives – continue to build on from Year 3

Subtract numbers mentally, including:

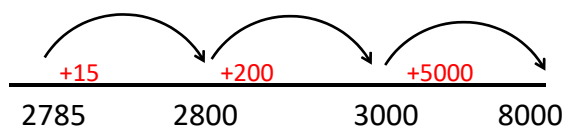
- a four-digit number subtract ones;
- a four-digit number subtract tens;
- a four-digit number subtract hundreds;
- a four-digit number subtract thousands.

Subtract numbers with up to 4 digits, using formal written method of columnar addition, where appropriate.

Mental strategies

- Using place value to partition and count back ones, tens and hundreds.
- Use number lines to support visualisations.

e.g. Find the difference by counting up $8000 - 2785 = 5215$



Written Strategies

- Build on year 3 methods for column addition using 4 digit numbers.

Year 5 objectives- continue to build on from Year 3 and 4

- Subtract numbers mentally, with increasingly large numbers.
- Subtract whole numbers with more than 4 digits, including using formal written method (column subtraction).

Mental strategies

- Using place value to partition and count back including decimals.
- Using estimating and rounding strategies.

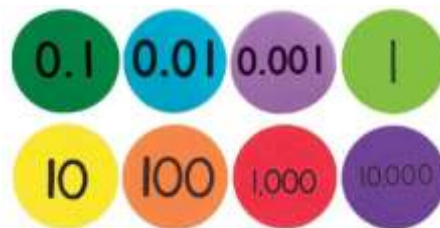
Written Strategies

Use the formal written method for the subtraction of decimal numbers:

$$725.75 - 233.82 = 491.93$$

$$\begin{array}{r} ^6 ^1 ^4 ^1 \\ \cancel{7} \cancel{2} \cancel{5} . 75 \\ - 233.82 \\ \hline 491.93 \end{array}$$

Place value counters could be used to support these calculations:



Ensure that the decimal points line up. Continue to use the language of place value to ensure understanding e.g. 5 hundredths subtract 2 hundredths.

Year 6 Objectives- continue to develop from learning previously taught

Perform mental calculations, including with mixed operations and large numbers.

Children should continue to practise and use formal written methods for larger numbers and decimals.

Our aim is that, by the end of Y6, children use mental methods (with jottings) when appropriate, but for other calculations, they use an efficient, formal written method accurately and with confidence.

Multiplication

Models and Images

number tracks
bead strings
number lines [marked and unmarked]
Base 10
place value counters
place value (arrow) cards
ten frames
Numicon
counting sticks
hundred squares
bar model
IT resources

Key Vocabulary

multiply
multiplication
times
lots of
groups of
sets of
product
multiple
double
factors
repeated addition
commutative

Fluency

TTRS

counting in different numbers in a line
double number of children in class

make arrays in PE

How many pencils will I need if every child in
class needs 3?

Year 1 Objectives

- Count in multiples of twos, fives and tens (to the 10th multiple).
- Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Mental Strategies

- Make Equal Groups:

Can you make the cubes into towers of 2?

Put the teddy bears into groups of 3.



- Count repeated groups of the same size in practical contexts to understand multiplication as repeated addition:



2 groups of 10 = 20

$10 + 10 = 20$

Double 10 is 20.

10 groups of 2 = 20

$2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 20$

Three groups of five.

How many altogether?

5, 10, 15

$2 + 2 + 2 = 6$

$3 + 3 = 6$

2 rows of 3 = 6

3 rows of 2 = 6

- Count in 2s, 5s and 10s, using a variety of concrete resources and contexts.



- Doubling - Use concrete and pictorial representations to calculate doubles to 10:



'double 2 is 4' $2 + 2 = 4$ or $4 = 2 + 2$



'double 3 is 6' $3 + 3 = 6$ or $6 = 3 + 3$

Year 2 Objectives- continue to build from year 1

- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication ($\times$), division ($\div$) and equals ($=$) signs.
- Show that the multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods and multiplication and division facts, including problems in context.

Mental Strategies

- Count in 2s, 3s, 5s, 10s**

Count in steps, using a variety of concrete resources.

- Use arrays to support multiplication**

Give children opportunities to explore, make and describe arrays.



$$6 + 6 = 12$$

$$6 \times 2 = 12$$

$$2+2+2+2+2+2=12$$

$$2 \times 6 = 12$$



$$4+4+4+4+4+4=24$$

$$4 \times 6 = 24$$

$$6 + 6 + 6 + 6 = 24$$

$$6 \times 4 = 24$$



$$5 \times 3 = 15$$



$$3 \times 5 = 15$$

Rotate arrays to show that multiplication of two numbers can be done in any order
(commutative law)

- Use arrays to support with calculating unknown facts, from known facts by partitioning numbers to create equivalent calculations (distributive law): e.g.



$$7 \times 5 = 6 \times 5 + 1 \times 5$$

$$6 \times 5 = 30$$

$$1 \times 5 = 5$$

$$7 \times 5 = 35$$

Children will use a range of vocabulary to describe multiplication and use practical resources, pictures, diagrams and the x and = signs to record.

- **Repeated Addition**



Children should be shown the different ways this picture can be recorded using words, numbers and signs.

‘There are 3 groups of 10 pens. How many pens altogether?’

‘ $10 + 10 + 10 = 30$ ’

‘3 groups of 10’

‘3 times ten’

‘3 lots of ten’

‘ $3 \times 10 = 30$ ’

‘ $10 \times 3 = 30$ ’

- Use the bar model to help children to visualise the concept of multiplication.

30		
10	10	10

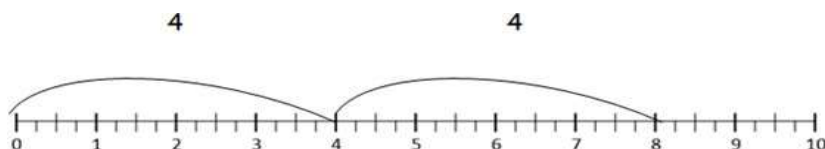
Year 3 Objectives- continue to build from Year 2

- Recall and use multiplication facts for the 3, 4 and 8 multiplication tables (continue to practise the 2, 5 and 10 multiplication tables).
- Count in steps of 4, 8, 50 and 100.
- Write and calculate mathematical statements for multiplication, using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental strategies and progressing to a formal written method.
- Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems, in which n objects are connected to m objects.

Mental Strategies

- Count in steps of 4, 8, 50 and 100**

Oral counting, supported by images, as appropriate. e.g. colour the multiples on a 1-100 grid; use hops along a landmarked number line.



Make links between 2s, 4s and 8 times tables; 3s and 6 times tables.

Use the knowledge of the commutative law to assist with learning times tables facts.

e.g. if they know $5 \times 8 = 40$, they know $8 \times 5 = 40$ too.

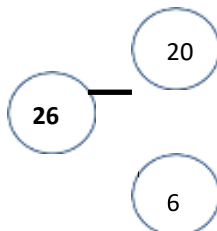
- Doubling**

Double numbers to 50 by partitioning into tens and ones, doubling and recombining e.g. to double 26, first partition into 20 and 6, then:

DOUBLE 20 = 40

DOUBLE 6 = 12

$40 + 12 = 52$



52	
26	26

- Use known facts**

Multiply multiples of 10 by single digit numbers, using known number facts; links to place value

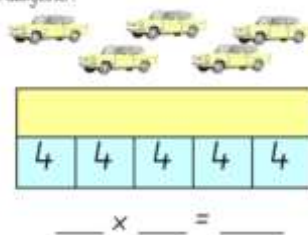
e.g. $3 \times 8 = 24$ therefore $30 \times 8 = 240$

Partition teen numbers to multiply by a single digit number (distributive law)

e.g. 14×3 becomes $10 \times 3 + 4 \times 3$

- Use bar modelling to solve problem

Use the bar model to calculate how many wheels there are altogether.

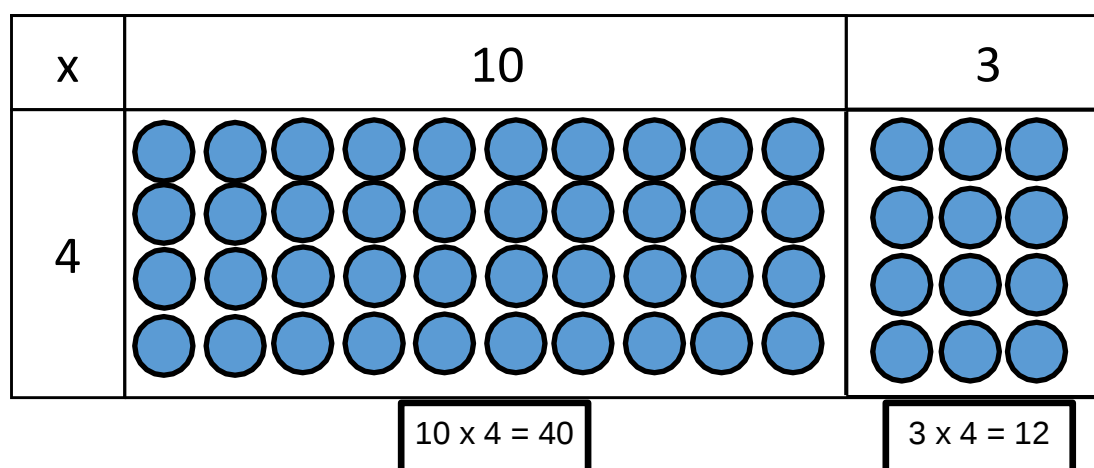


Written Strategies

Grid method

Use arrays to demonstrate partitioning into the multiples of ten and one:

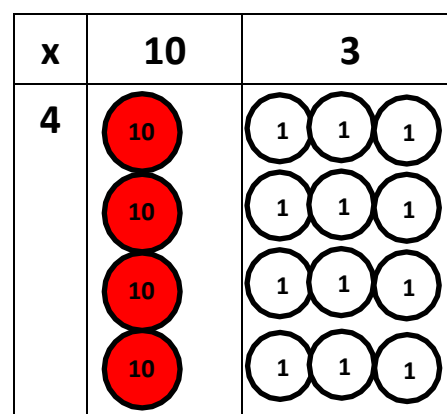
concrete



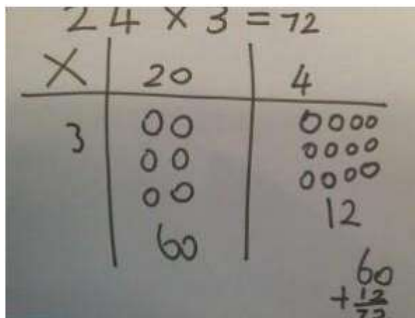
13×4 is partitioned into 10×4 and 3×4 .

The two parts should be added together to find the total.

- Counters can be replaced by Base 10 for efficiency.
- Make links between using Base 10 materials and place value counters.
- Once the concept of partitioning multiples is secure, children can move from using equipment to writing numbers.



pictorial



abstract

x	10	3
4	40	12

$$10 \times 4 = 40$$

$$3 \times 4 = 12$$

$$40 + 12 = 52$$

Expanded column method:

$$\begin{array}{r}
 13 \\
 \times 4 \\
 \hline
 12 \quad (3 \times 4) \\
 + 40 \quad (10 \times 4) \\
 \hline
 52
 \end{array}$$

Year 4 Objectives- continue to build on from Year 3

- Recall multiplication and division facts for multiplication tables up to 12×12 .
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers.
- Recognise and use factor pairs and commutativity in mental calculations.
- Multiply two-digit and three-digit numbers by a one-digit number, using a formal written layout.
- Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems, such as: n objects are connected to m objects.

Build on Year 3 strategies.

Mental Strategies

- Use concrete resources to illustrate commutativity and factor pairs.

eg. $3 \times 7 = 7 \times 3 = 21$

- Count in steps of 6, 7, 9, 25 and 1000.

Oral counting forwards and backwards, using apparatus, links to prior learning and commutativity as appropriate.

- Use number facts.

Learn the multiplication facts to 12×12 :

A multiplication square can be useful for finding commutative facts, spotting easy patterns and identifying tricky facts that need to be learnt.

- Use place value knowledge.

Multiply whole numbers and decimals to 1dp by 10, 100 and 1000, using understanding of place value and other known multiplication facts.

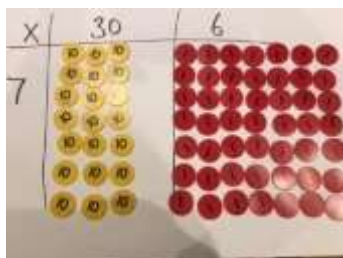


×	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Written Methods

Children should be progressing onto formal written methods for appropriate calculations. However, the use of the grid method along with concrete apparatus to support this, is strongly recommended and is appropriate to enhance a greater depth of understanding.

- **Grid Method**



X	30	6
7	210	42

Remember to add the partial products to find the total $210 + 42 = 252$

- **Expanded column method**

$$\begin{array}{r} 36 \times 7 = \\ 36 \\ \times 7 \\ \hline 42 \quad (7 \times 6) \\ 210 \quad (7 \times 30) \\ \hline 252 \end{array}$$

- **Compact column method**

$$\begin{array}{r} 36 \times 7 = \\ 4 \\ 36 \\ \times 7 \\ \hline 252 \end{array}$$

Multiply 6 by 7 to get 42. Place the 2 in the ones column and the 4 (4 tens) above the tens column.

Multiply 3 (3 tens) by 7 to get 21. Place these in the remaining columns, remembering to add the 4 tens to the 1.

N.B Continue to reinforce the children's understanding of place value by ensuring they recognise that the 3 is worth 30 and the 21 is worth 210.

Year 5 Objectives- continue to build on from Year 4

- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.
- Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- Establish whether a number up to 100 is prime and recall prime numbers up to 19.
- Multiply numbers mentally, drawing upon known facts.
- Multiply whole numbers and those involving decimals by 10, 100 and 1000.
- Recognise and use square numbers and cube numbers, as well as the notation for squared (²) and cubed (³).
- Multiply numbers up to 4 digits by a one- or two-digit number, using a formal written method, including long multiplication for two-digit numbers.
- Solve problems involving multiplication, including using their knowledge of factors and multiples, squares and cubes.
- Solve problems involving multiplication, including scaling by simple fractions and problems involving simple rates.

Mental Strategies

Children should continue to use a range of mental strategies to support their work, including use of doubles and halves, number facts and place value knowledge.

• Near multiples

Multiply using near multiples by rounding e.g. 32×29 becomes $(32 \times 30) - 32$

• Partitioning

Use partitioning to multiply two-digit and three-digit numbers by a single-digit number e.g. 402×6 becomes 400×6 and 2×6 .

Use partitioning to multiply decimals by a single-digit number e.g. 4.5×3 becomes 4×3 and 0.5×3 .

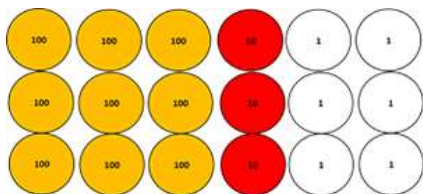
• Understanding of factors and multiples

Use knowledge of factors and multiples in multiplication e.g.

$$35 \times 4 = 7 \times 5 \times 2 \times 2 \rightarrow \text{this could be solved as } 7 \times 2 \times 10 \text{ or } 70 \times 2 \text{ etc.}$$

Written Methods

Build on the work covered in Y4.



x	300	10	2
3	900	30	6

Add the partial products $900 + 30 + 6 = 936$

$$\begin{array}{r}
 312 \\
 \times 3 \\
 \hline
 6 \quad (3 \times 2) \\
 30 \quad (3 \times 10) \\
 900 \quad (3 \times 300) \\
 \hline
 936
 \end{array}$$

- Expanded long multiplication (two-digit numbers multiplied by a teen number):

$$23 \times 13 = 299$$

$$\begin{array}{r}
 23 \\
 \times 13 \\
 \hline
 9 \quad (3 \times 3) \\
 60 \quad (20 \times 3) \\
 30 \quad (3 \times 10) \\
 200 \quad (20 \times 10) \\
 \hline
 299
 \end{array}$$

X	20	3
10	200	30
3	60	9

Add the partial products, using the most appropriate method.

$$200 + 60 + 30 + 9 = 299$$

Use the language of place value as the method is demonstrated:

First, multiply 3 by 3 to give 9.
 Next, multiply 20 by 3 to give 60.
 Now, multiply 3 by 10 to give 30.
 Finally, multiply 20 by 10 to give 200.
 Add the partial products up.

- **Compact long multiplication (formal method)**

$24 \times 16 =$ is shown below as an expanded method and then in compact form:

$ \begin{array}{r} 24 \\ \times 16 \\ \hline 24 \quad (4 \times 6) \\ 120 \quad (20 \times 6) \\ 40 \quad (4 \times 10) \\ 200 \quad (20 \times 10) \\ \hline 384 \end{array} $	→	$ \begin{array}{r} 24 \\ \times 16 \\ \hline 144 \\ + 240 \\ \hline 384 \end{array} $
---	---	---

Use the language of place value as the method is demonstrated:

First, multiply 4 by 6 to give 24; record the 4 in the units column and carry the 20 as 2 tens in the tens column.

Next, multiply 20 by 6 to give 120; remember to add the 2 tens carried $120 + 20 = 140$. Record 4 tens in the tens column and 1 hundred in the hundreds column.

Next, write a zero in the ones column because you are multiplying by 10 (so the product will end in a zero which will be used as a place holder).

Then, multiply 4 by 10 to give 40; record a 4 in the tens column.

Now, multiply 20 by 10 to give 200 which is recorded with a 2 in the hundreds column.

Then add the 2 partial products together.

When children are confident with compact multiplication, extend with three-digit numbers multiplied by two-digit numbers.

124×26 becomes

	1	2	
	1	2	4
×		2	6
	1	1	
		7	4
+	2	4	8
		7	4
	3	2	2
			4

Year 6 Objectives- continue to build on from previous years

- Perform mental calculations, including with mixed operations and large numbers.
- Multiply one-digit numbers, with up to two decimal places, by whole numbers.
- Identify common factors, common multiples and prime numbers.
- Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.
- Solve problems involving addition, subtraction, multiplication and division.
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

Mental Strategies

Children should continue to use a range of mental strategies to support their work, including use of doubles and halves, number facts and place value knowledge.

Written Methods

- **Formal written method of long multiplication for decimals**

$$53.2 \times 24$$

$$\begin{array}{r} 1 \\ 53.2 \\ \times 24.0 \\ \hline 212.8 \\ 1064.0 \\ \hline 1276.8 \end{array}$$

Use the language of place value as the method is demonstrated.

Provide a wide range of contexts, including the use of scaling, units of measure and large whole numbers, as well as decimals to apply the skills. Also, include problem solving which includes more than one operation.

By the end of Y6, children use mental methods (with jottings) when appropriate, but for calculations that they cannot do mentally, they use an efficient, formal written method accurately and with confidence.

Division

Models and Images

practical objects (e.g. socks, straws, cubes, hoops, pots)

Bar model

PV counters

Diennes

counting stick

IT Resources

Key Vocabulary

share

divide

lots of

groups of

sets of

halving

equally

remainders

inverse

quotient

divisor

dividend

repeated subtraction

Fluency

Sort children into groups of 2, 5 ect: How many groups? How many left over?

Divide stationary between children.

Year 1 Objectives

- Solve one-step problems involving division by calculating the answer, using concrete objects, pictorial representations and arrays with the support of the teacher.
- Count in multiples of twos, fives and tens (to the 10th multiple).

Mental Strategies

Children will start with practical sharing, using a variety of resources.

They will share objects into equal groups, in a variety of situations.

They will begin to use the vocabulary associated with division in practical contexts.

Make sure children understand what equal and unequal means.

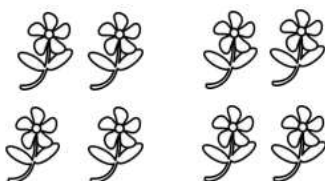
It is important that both concepts of division are introduced and understood, alongside the relevant language. There must be sufficient opportunities to manipulate practical resources, in order to support the learning of the difference between the concepts [grouping and sharing].

Sharing

concrete



pictorial



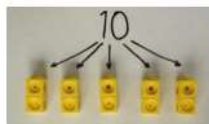
abstract

What is 8 shared between 2 people?

Grouping

concrete

Divide quantities into equal groups.
Use cubes, counters, objects or place value counters to aid understanding.



pictorial



abstract

Divide 10 into 5 groups. How many are in each group?



Halving

Find half of even numbers up to 12, using concrete apparatus. Explore what happens when an odd number is halved.

Year 2 Objectives – continue to build on from Year1

- Recall and use division facts for the 2, 5 and 10 multiplication tables.
- Solve problems involving division, using materials, arrays, repeated subtraction, mental methods, and multiplication and division facts, including problems in contexts.
- Calculate mathematical statements for division within the multiplication tables and write them using the division ($\div$) and equals (=) signs.
- Show that division of one number by another is not commutative [i.e. can be done in any order].

Mental Strategies

Children will use a range of vocabulary to describe division and use practical resources, pictures, diagrams and the division ($\div$) and equals (=) signs to record, using multiples that they know.

Halving

Find half of even numbers up to 40. Explore what happens when halving an odd number.

Begin to know the halves of multiples of 10, up to 100 (e.g. half of 70 is 35).

Make links to fractions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$. Teach explicitly the links between fractions and division (e.g. half is the same as dividing by 2; a third is the same as dividing by 3 and a quarter as dividing by 4).

Number facts

Learn the division facts for 2x, 5x and 10x tables, using relevant vocabulary:

10 divided by 5

10 shared between 5

10 grouped into 5's

Use of arrays

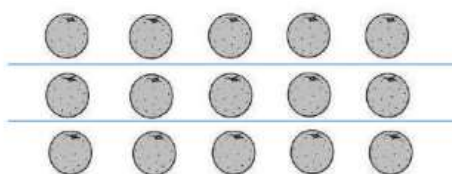
concrete

Link division to multiplication by creating an array and thinking about the number sentences that can be created.



Eg $15 \div 3 = 5$ $5 \times 3 = 15$
 $15 \div 5 = 3$ $3 \times 5 = 15$

pictorial



abstract

Find the inverse of multiplication and division sentences by creating four linking number sentences.

$5 \times 3 = 15$
 $3 \times 5 = 15$
 $15 \div 5 = 3$
 $15 \div 3 = 5$

Year 3 Objectives- continue to build on from Year 2

- Recall and use division facts for the 3, 4 and 8 multiplication tables.
- Write and calculate mathematical statements for division, using the multiplication tables that they know, including for 2 digit numbers divided by single-digit numbers, using mental strategies and progressing to formal written methods.
- Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems, in which n objects are connected to m objects.

Mental Strategies

Grouping

Recognise that division is not commutative (e.g. $16 \div 8$ does not equal $8 \div 16$)

Relate division to multiplication (i.e., they are the inverse of each other):

$$? \times 5 = 30 \text{ is the same as } 30 \div 5 = ?$$

Divide multiples of 10 by single-digit numbers e.g. $240 \div 8 = 30$ because we know $24 \div 8 = 3$

Use number facts

Learn the division facts for 3x, 4x, 5x, 8x, and 10x tables, using a range of division vocabulary

16 divided by 4

16 shared between 4

16 grouped into 4's

Divide larger numbers mentally by partitioning into multiples of the divisor:

Use arrays to support e.g. $39 \div 3$ becomes



$$30 \div 3 = 10$$

$$10 \times 3 = 30$$



$$9 \div 3 = 3$$



Use division facts to find unit and simple non-unit fractions of amounts

e.g. $\frac{3}{4}$ of 12 becomes



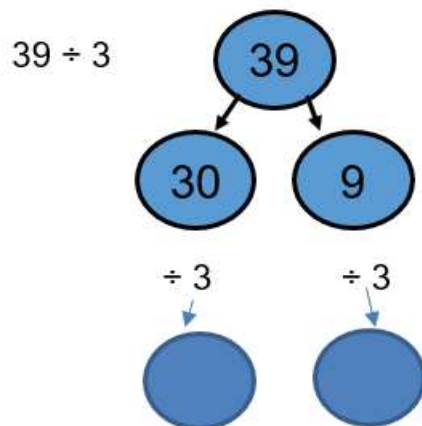
$$12 \div 4 = 3$$

$$3 \times 3 = 9$$



Written Strategies

Encourage children to use their times tables knowledge:

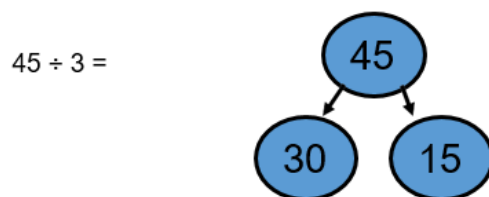


$$\begin{array}{r} 13 \\ 3 \overline{) 39} \end{array}$$

List multiples
of 3
0
3
6
9

Grouping

Divide two-digit numbers by a single-digit beyond the children's times table knowledge:



Use times table knowledge to support calculations:

$$10 \times 3 = 30$$

$$5 \times 3 = 15$$

$$10 + 5 = 15 \text{ [there are 15 groups of 3 in 45]}$$

Year 4 Objectives- continue to build on from Year 3

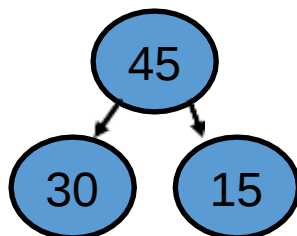
- Recall division facts for multiplication tables up to 12×12 .
- Use place value, known and derived facts to divide mentally, including dividing by 1.
- Recognise and use factor pairs in mental calculations.
- Divide two-digit and three-digit numbers by a one-digit number, using formal written layout.
- Solve problems involving division, integer scaling problems and harder correspondence problems, such as, n objects are connected to m objects.

Mental Strategies

Grouping

Divide two-digit numbers by a single-digit beyond the children's times table knowledge:

$$45 \div 3 =$$



Use times table knowledge to support calculations:

$$10 \times 3 = 30$$

$$5 \times 3 = 15$$

$$10 + 5 = 15 \text{ [there are 15 groups of 3 in 45]}$$

Divide multiples of 100 by single-digit numbers, e.g. $2400 \div 8 = 300$ because we know $24 \div 8 = 3$

Written Strategies

To write and calculate mathematical statements for division, using the multiplication tables that the children know:

$$32 \div 8 = 4$$

short division

Use the language of place value throughout and make the link to partitioning.

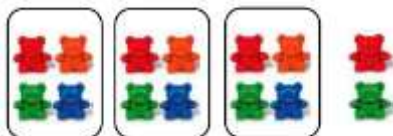
$$\begin{array}{r} 14 \\ 7 \overline{) 98} \end{array}$$

$$\begin{array}{r} 14 \text{ quotient} \\ 8 \text{ divisor} \overline{) 112 \text{ dividend}} \end{array}$$

Short division with remainders

concrete

$14 \div 3 =$
Divide objects between groups and see how much is left over



pictorial

Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.



Draw dots and group them to divide an amount and clearly show a remainder.



abstract

$$\begin{array}{ccccccc} 29 \div 8 = 3 \text{ REMAINDER } 5 \\ \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \\ \text{dividend} \quad \text{divisor} \quad \text{quotient} \quad \text{remainder} \end{array}$$

$$\begin{array}{r} 13 \text{ r}5 \\ 6 \overline{) 83} \end{array}$$

List multiples
0
6
12
18
24

Year 5 Objectives- continue to build on from Year 4

- Divide numbers up to 4 digits by a one-digit number, using the formal written method of short division and interpret remainders appropriately for the context.
- Solve problems involving division, including using their knowledge of factors and multiples, squares and cubes.
- Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign.
- Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

Mental Strategies

Use the multiples of 10 to support partitioning:

e.g. $186 \div 3$ becomes $30 \times 6 = 180$ and $1 \times 6 = 6$ therefore the quotient is 31

Use number facts

- Use division facts from times tables to divide multiples of powers of 10 of the divisor:
 $3600 \div 9 = 400$ using $36 \div 9 = 4$
- Use knowledge of multiples and factors, also tests for divisibility, in mental division e.g.
 $246 \div 6 = 123 \div 3$
- We know that 525 is divisible by 5 because it ends in a 5 and all multiples of 5 end in either a 5 or a 0
- 525 is divisible by 3 because the sum of its digits is divisible by 3 ($5 + 2 + 5 = 12$)
- Divide whole numbers by 10, 100 and 1000 to give whole number answers, or answers with up to 3 decimal places.

Written Strategies

Continue to practise the formal written method of short division with whole number answers:

$$184 \div 8 = 23 \quad \longrightarrow \quad \begin{array}{r} 23 \\ 8 \overline{) 184} \\ \underline{16} \\ 24 \\ \underline{24} \\ 0 \end{array} \quad \begin{array}{|c|} \hline \text{List} \\ \text{multiples of} \\ 8 \\ \hline \end{array}$$

Continue to use the language of place value to ensure understanding.

Introduce remainders e.g. $432 \div 5 = 86 \text{ r } 2$

How many 5s are there in 400? Use 5 times table knowledge that $5 \times 8 = 40$, therefore $5 \times 80 = 400$. Record with an 8 in the tens column.

Next, how many 5s in 30? Again, use times tables to help $5 \times 6 = 30$. Record with a 6 in the ones column.

Finally, how many 5s in 2? This doesn't work, so the 2 is recorded as a remainder.

$$\begin{array}{r} 86 \text{ r } \frac{2}{5} \\ 5 \overline{) 432} \\ \underline{40} \\ 32 \\ \underline{30} \\ 2 \end{array}$$

The remainder can also be expressed as a fraction: $\frac{2}{5}$ which is the remainder divided by the divisor.

Year 6 Objectives – continue to build on from previous years

- Perform mental calculations, including with mixed operations and large numbers.
- Divide numbers up to 4 digits by a two-digit whole number, using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context.
- Divide numbers up to 4 digits by a two-digit number, using the formal written method of short division and where appropriate, interpreting remainders according to the context.
- Solve problems involving addition, subtraction, multiplication and division.
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.
- Use written division methods in cases where the answer has up to two decimal places.

Mental Strategies

Grouping

Use multiples of 10 of the divisor to divide large numbers:

e.g. $378 \div 9$ becomes $40 \times 9 = 360$ and $2 \times 9 = 18$ therefore the quotient is 42

Use number facts

Use division facts from times tables to divide decimal numbers by single-digit numbers:

$2.4 \div 6 = 0.4$ using $24 \div 6 = 4$

Identify and use common factors, common multiples and prime numbers in mental division:

$288 \div 24 = 144 \div 12$

Use tests for divisibility to aid mental calculation:

Divide whole numbers by 10, 100, 1000 and 10,000 to give whole number answers, or answers with up to 3 decimal places.

Know and use equivalence between simple fractions, decimals and percentages including in different contexts.

Recognise a given ratio and reduce a given ratio to its simplest form.

Written Strategies

Continue to practise the formal method of short division, with and without remainders, using the language of place value to ensure understanding (see Y5 guidance).

Formal method of long division

When the divisor is > 12 , short division is not appropriate and long division must be used, which involves the repeated subtraction of multiples of the divisor.

How many 15s are there in 400? I know that $15 \times 10 = 150$ so $15 \times 20 = 300$. So I will subtract 300 from 432 and record the 20 as 2 tens by placing a 2 in the tens column.

Next, I will subtract 300 from 432, which leaves 132. I can't use 15×10 because it is bigger than 132. I know I need to multiply 15 by an even number to give me a multiple of 10, so I will try $15 \times 8 = 120$, which is lower than 132 so I can use it. To record it, I put 8 in the ones column and subtract 120 from 132, which leaves 12.

12 is smaller than 15, so there are no more groups of 15 therefore, 12 is the remainder.

$$\begin{array}{r} 28 \text{ r } 12 \\ 15 \overline{) 432} \\ \underline{300} \\ 132 \\ \underline{120} \\ 12 \end{array}$$

Remember the remainder can be expressed as a fraction, which should be reduced to its lowest form, or as a decimal.

$432 \div 15$ becomes

$$\begin{array}{r} 28 \\ 15 \overline{) 432} \\ \underline{300} \\ 132 \\ \underline{120} \\ 12 \end{array} \quad \begin{array}{l} 15 \times 20 \\ 15 \times 8 \end{array}$$

The r12 is converted to a decimal, using the knowledge that $\frac{1}{5} = 0.2$ therefore $0.2 \times 4 = 0.8 = \frac{4}{5}$

$$\frac{12}{15} = \frac{4}{5}$$

When children are confident, they can learn this alternative method of recording long division, which expresses the remainder as a decimal, without the need to convert from a fraction.

$432 \div 15$ becomes

$$\begin{array}{r} 28.8 \\ 15 \overline{) 432.0} \\ \underline{30} \downarrow \\ 132 \\ \underline{120} \downarrow \\ 120 \\ \underline{120} \\ 0 \end{array}$$

Children will need to select the most effective method for each calculation/problem they meet. For example, division involving money will use decimals.

Answer: 28.8